Off-Road Cycling Facilities - Sight Triangle Requirements at Stop Controlled Driveway and Side Road Crossings

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Introduction

Increasingly local and regional municipalities are constructing bicycle facilities within the boulevard – i.e. the space between the roadway and the property line, whether they be dedicated cycling tracks or multi-use pathways (MUP's). In addition, cyclists also often use sidewalks. Cyclists can travel in excess of 20 km/h, and where these cycling facilities cross stop controlled driveways and side roads there is a significant potential for conflict between cyclists and exiting drivers, especially considering that many drivers are 'non-compliant' and do not always stop in advance of the cycling facility. Therefore such 'non-compliant' drivers and cyclists should ideally have enough sight distance to allow them time to observe, initiate and execute the desired actions to avoid a collision. This paper will show that there cannot be a 'one-size-fit-all' solution because sight triangle requirements are a function of several variables that can differ from location to location. It is therefore imperative that required sight triangle dimensions be calculated on a site by site basis. The paper will develop and present the equations and a methodology that can be used to calculate the ***minimum*** and ***desirable*** sight triangle dimensions for any location that account for "compliant" and "non-compliant" drivers respectively.

State of Practice

Although North American Guidelines, such as for example TAC's *Geometric Design Guide for Canadian Roads* (2017), AASHTO's *Guide to Bicycle Facilities* (2012), FHWA's *Separated Bike Planning and Design Guide* (2015) all acknowledge the importance of providing clear sight lines between cyclists and driveway/side road vehicles, none provide for an analytical methodology to calculate required sight triangle dimensions that takes into account critical variables such as the type of facility, driver behaviour, and design parameters of the facility.

Types of Off-Road Cycling Facilities

There are three types of off-road facilities within the boulevard that can be used by cyclists:

i) Multi-Use Pathways (MUP's)

MUP's are designed to be shared by pedestrians, cyclists and other forms or active transportation such as in-line skating, skateboarding, scootering, etc. From a geometric design perspective cyclists are the design user because they operate at higher speeds than other users. In general MUP's are located in close proximity to the property line, and at driveways and side roads they either run straight through or can '***bend in'*** towards the main road. See ***Appendix A***.

ii) In-Boulevard Cycle Tracks

In-boulevard cycle tracks are for cyclist use only and are generally located close (1 to 1.5 meters) to the edge of the main roadway. To prevent outbound driveway/side road vehicles from blocking the cycle track, and to ensure good visibility between inbound right-turn vehicles from the main roadway and cyclists a '***bend out***' design is recommended. See ***Appendix A***.

iii) Sidewalks

The Highway Traffic Act (HTA) of Ontario does not expressly forbid cyclists from using sidewalks. Instead, local municipalities in Ontario have adopted their own by-laws to regulate cyclists' use of sidewalks. For example, the City of Toronto has a by-law that no person older than 14 years old may use a bicycle on a sidewalk. Even in Provinces like British Columbia[1] and Quebec[2] where sidewalk use by cyclists is prohibited, there are provisions that would, under certain circumstances, allow cyclists to legally use a sidewalk. It is therefore prudent to consider a sidewalk also as a cycling facility.

Sight Triangle Requirements

There are two types of sight triangles to consider. These, and their associated dimensions, are illustrated in **Figure 1.**

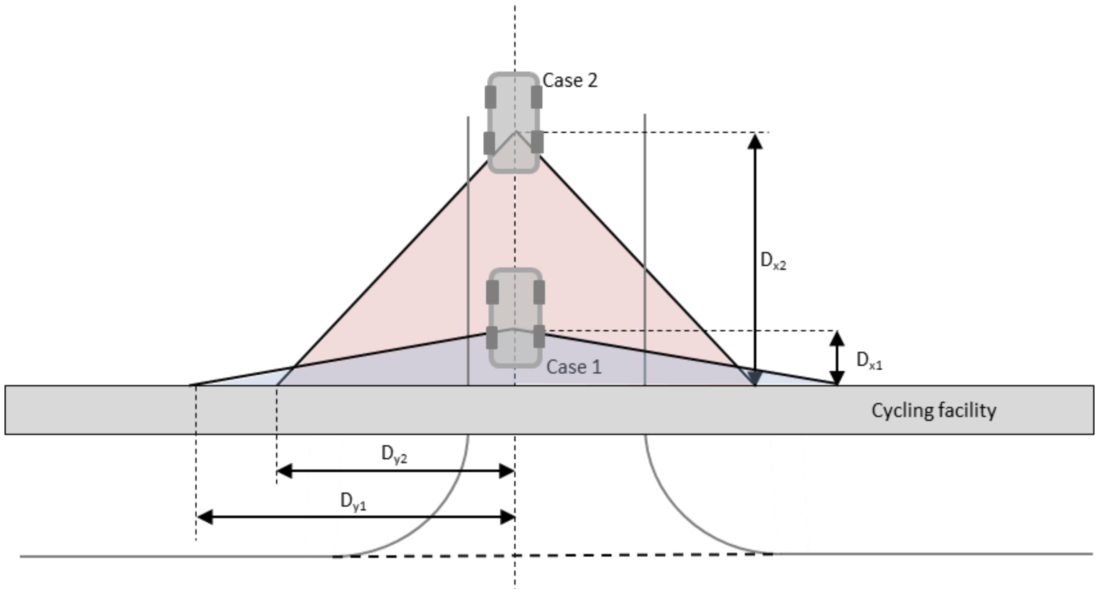

Figure 1 : Sight Triangle Requirements and Dimensions

The dimensions of each sight triangle are defined by two dimensions, $\mathbf{D_x}$ and $\mathbf{D_y}$.

- $\mathbf{D_x}$ is the distance between the edge of the cycling facility and the driver's eye
- $\mathbf{D_y}$ is the distance from the front of the bicycle to the centre of the vehicle's trajectory

Ideally $\mathbf{D_y}$ should be measured from the trajectory of the driver's eye – however measuring from the centre of the vehicle is a simplification that is required to enable $\mathbf{D_y}$ from being measured from a common point in both directions.

i) <u>**Case 1 : Absolute Minimum Sight Triangle**</u>

The driver approaches the cycling facility and stops in advance of it. After looking for a suitable gap on the cycling facility the driver then crosses and clears the cycling facility, without the cyclist having to brake and reduce speed. This case represents the ***absolute minimum*** sight triangle requirement and should be present at each crossing location. As cyclists have the right-of-way where cycling facilities cross driveways/side roads they should not have to do emergency stops – although the minimum required stopping sight distance should always be available.

[1] Motor Vehicle Act (RSBC 1996), Section 183(2)a
[2] Highway Safety Code, Section 492.1

ii) Case 2 : Desirable Sight Triangle

The driver approaches the cycling facility with no intention to stop in advance of it, but aims to stop at the edge of the main road instead. Ideally there should be enough sight distance for such a driver, once a cyclist is observed, to perform an emergency stop in advance of the cycling facility, or for a cyclist to make an emergency stop. This case represents the ***desirable*** sight triangle requirement. In practice, a fully unencumbered desirable sight triangle will be difficult to achieve in most cases. The reason being that it is likely that a large portion of the sight triangle will be over private property, over which a road authority have little to no control. However, as this paper will show, the road authority has the ability to modify the cycling facility's design parameters to decrease the required size of the desirable sight triangle, and the risk of it being encumbered by features on private property.

Determining Sight Triangle Dimensions

Determining the values of $\mathbf{D_{y1}}$, $\mathbf{D_{y2}}$, $\mathbf{D_{x1}}$ and $\mathbf{D_{x2}}$ (as illustrated in **Figure 1**) requires a kinematic analysis using Newton's equations of motion. It is assumed that the cyclist is travelling at a constant speed approaching the driveway and is unprepared to stop since in all cases they will have the right of way.

i) Case 1 – The Compliant Driver

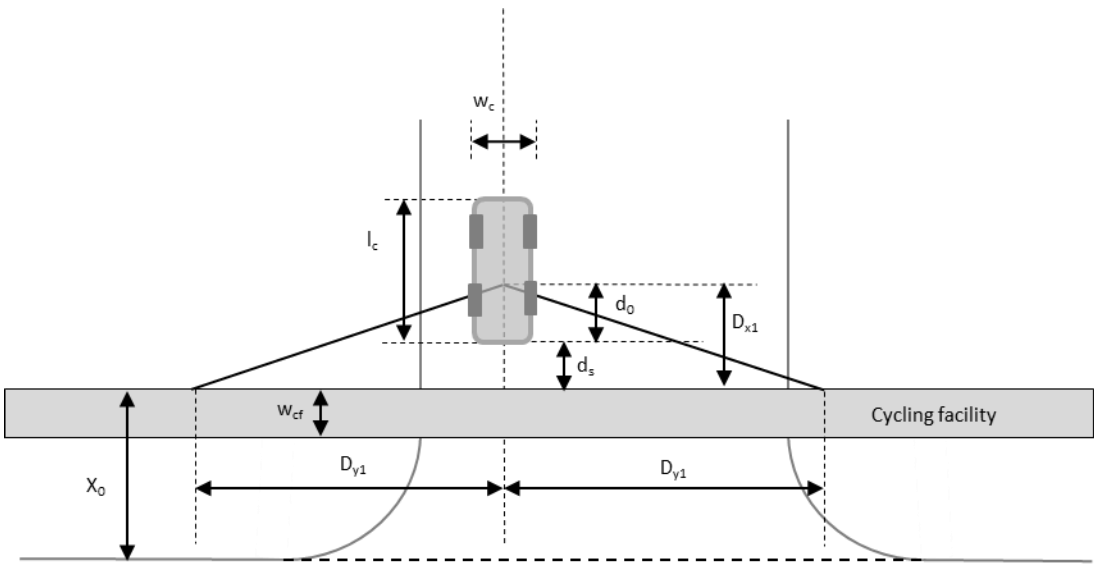

Figure 2 : Case 1 – Illustration of Equation Parameters

$\mathbf{D_{x1}}$ in this case is simply given by the distance of the stopped driver's eye to the closest edge of the cycling facility.

$$D_{x1} = d_s + d_0 \qquad (1)$$

Where

d_s = *Distance from the front of vehicle to edge of the cycling facility (m)*
d_0 = *Distance from the front of vehicle to the driver's eye (m)*

D_{y1} is given by the distance a cyclist is expected to travel in the time it would take the rear bumper of the initially stopped vehicle to completely clear the cycling facility.

$$D_{y1} = 0.278 t_{cl} V_c + 0.5 w_c \qquad (2)$$

Where

V_c = *Design speed of cyclist (km/h)*
t_{cl} = *Time for back of vehicle to clear the cycling facility (sec)*
w_c = *Width of design vehicle (m)*

To estimate the clearance time (t_{cl}) two sub-cases must be considered – these are discussed below. **Appendix B** shows the derivation of the equations to calculate the clearance time for each sub-case, as required by ***Equation 2*** to calculate D_{y1}.

1a) The vehicle fits completely between the main road and the cycling facility

In this sub-case the value of the offset X_0 is large enough that a vehicle can be fully accommodated between the main road and the cycling facility.

To cross the cycling facility, in the most conservative scenario, a driver will accelerate for a short period – and then decelerate again to stop before the curb.

The total distance the vehicle will travel from start to stop is given by D_T:

$$D_T = X_0 + d_s \qquad (3)$$

However, to clear the cycling facility the vehicle only has to travel a distance sufficient to get the back of the vehicle to clear the cycling facility. This clearance distance D_{cl} is given by:

$$D_{cl} = w_{cf} + d_s + l_c \qquad (4)$$

Where

w_{cf} = *Width of cycling facility (m)*
d_s = *Distance from front of vehicle to edge of the cycling facility (m)*
l_c = *Length of design vehicle (m)*

Figure 3 illustrates the scenario where the offset (X_0) is large enough such that the vehicle only has to start decelerating after the rear of the vehicle clears the cycling facility.

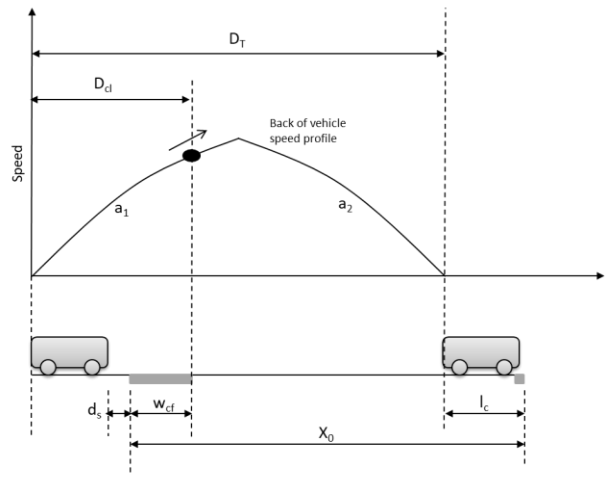

Figure 3 : Continuous acceleration over cycling facility

The clearance time t_{cl} for this scenario is given by:

$$t_{cl} = \sqrt{\frac{2D_{cl}}{a_1}} \qquad (5)$$

And applies where:

$$X_o \geq \left(\frac{a_1}{a_2} + 1\right) D_{cl} - d_s \qquad (6)$$

Where:

a_1 = *Normal acceleration rate – gradient adjusted [±9.81G] (m/s²)*
a_2 = *Normal deceleration rate - gradient adjusted [±9.81G] (m/s²)*[3]
G = *Grade (m/m)*

Assuming $\mathbf{a_1} = \mathbf{a_2}$ and conservative parameters $\mathbf{w_{cf}}$ = 1.5 m, $\mathbf{l_c}$ = 5 m, d_s = 0, ***Equation 5*** will only be applicable when X_0 > 13 m. Such large values of $\mathbf{X_0}$ are unlikely to occur in practice.

Figure 4 shows the scenario where the vehicle has to start decelerating before the rear of the vehicle clears the cycling facility.

[3] In all equations where required the deceleration rate (normal and emergency) is a positive value.

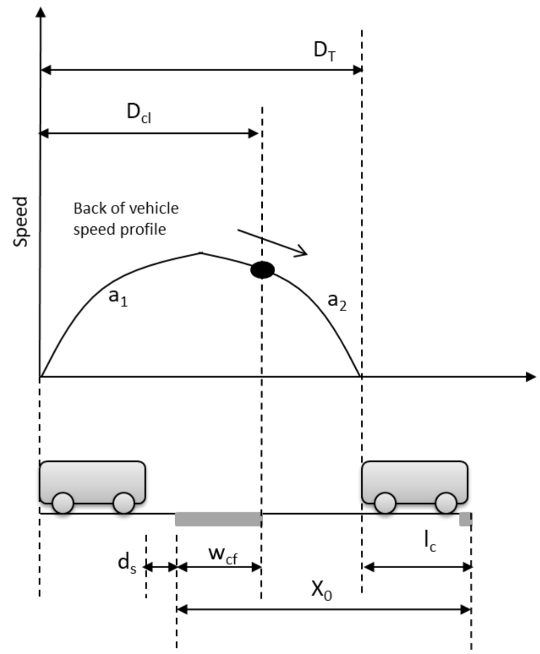

Figure 4 : Deceleration over part of cycling facility

The clearance time t_{cl} for this scenario is given by:

$$t_{cl} = 2\sqrt{\frac{D_T}{a_1}} - \sqrt{\frac{2(D_T - D_{cl})}{a_1}} \qquad (7)$$

Case 1(b) - The vehicle does not fit completely an part of it blocks the facility.

If the vehicle does not fit between the edge of the main road and the near edge of the cycling facility, partially crossing the cycling facility and stopping at the road edge will block a cyclist's path, forcing it to stop, or to perform a potentially unsafe detour around the back of the blockading vehicle.

In this case, the required $\mathbf{D_{y1}}$ should be the greater off:

- The minimum distance required by a cyclist to make an emergency stop (see ***Equation 14***)
- A $\mathbf{D_{y1}}$ calculated using ***Equation 2*** based on a clearance time for the scenario where the driver is able to safely accelerate directly into main road traffic without having to stop at the curb. For this scenario the clearance time ($\mathbf{t_{cl}}$) is given by the time it takes a vehicle to accelerate at rate ($\mathbf{a_1}$) over the clearance distance $\mathbf{D_{cl}}$ and is given by ***Equation 5***.

ii) **Case 2 – The non-compliant driver**

In this Case the driver is decelerating towards the cycling facility with no intention to stop in advance of it but aims to stop at the edge of the main road it instead. In this situation there should be enough sight distance for a driver, once a cyclist is observed, to perform an emergency stop in advance of the cycling facility.

- **Determining D_{x2}**

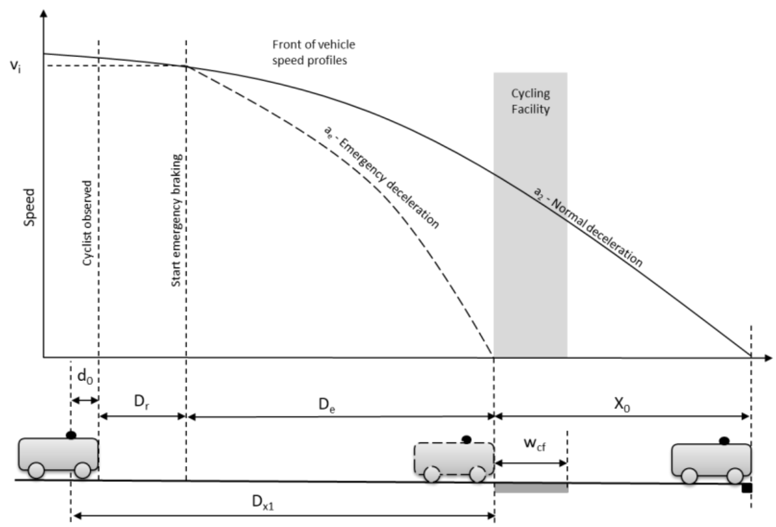

Figure 5 : Case 2 – Parameters and Variables to Calculate D_{x2}

A driver will approach the cycling facility with a normal declaration rate $\mathbf{a_2}$ and if a cyclist is observed it will decelerate from a speed $\mathbf{V_i}$ at an emergency deceleration rate of $\mathbf{a_e}$ – to stop over a distance $\mathbf{D_e}$.

In this case $\mathbf{D_{x1}}$ is given by the sum three components:

$$D_{x1} = d_0 + D_r + D_e \qquad (8)$$

Where:

d_0= *Distance between front of vehicle to driver's eye (m)*
D_r = *Distance travelled by driver during perception-reaction time period* $\mathbf{t_r}$ *(m)*
D_e = *Distance required to make an emergency stop in advance of cycling facility (m)*

Appendix C shows the derivation of the equations to calculate $\mathbf{D_e}$ and $\mathbf{D_r}$.

The equation for $\mathbf{D_e}$ is :

$$D_e = \frac{X_0 a_2}{a_e - a_2} \qquad (9)$$

Where:

a_2 = *Normal deceleration rate - gradient adjusted [±9.81G] (m/s²)*
a_e = *Emergency deceleration rate - gradient adjusted [±9.81G] (m/s²)*
X_0 = *The distance between curb and far edge of cycling facility (m)*
G = *Gradient (m/m)*

If we assume $\mathbf{a_2}$ and $\mathbf{a_e}$ to be constants then there is a direct linear relationship between the emergency stopping distance ($\mathbf{D_e}$) and the distance of the cycling facility from the curb ($\mathbf{X_0}$).This means that the further the cycling facility is located from the curb, the higher the emergency stopping distance ($\mathbf{D_e}$) and ultimately the dimension of the required daylight triangle along the approach ($\mathbf{D_{x2}}$).

The equation for $\mathbf{D_r}$ is:

$$D_r = \sqrt{2a_eD_e}t_r + 0.5a_2t_r^2 \qquad (10)$$

Where

t_r = *Driver's perception-reaction time (sec)*

Substituting ***Equation 10*** into ***Equation 8*** gives:

$$D_{x2} = d_0 + \sqrt{2a_eD_e}t_r + 0.5a_2t_r^2 + D_e \qquad (11)$$

- **Determining $\mathbf{Dy_2}$**

The total time required to make an emergency stop from the moment of a cyclist is observed until coming to a stop in advance of the cycling facility is given by:

$$t_s = t_e + t_r \qquad (12)$$

Where:

*t_e = Time to make an emergency stop – from **Equation B5** in **Appendix C***
t_r = Driver's perception-reaction time (sec)

The distance the cyclist travel at normal speed over time $\mathbf{t_s}$ is given by:

$$D_{nc} = 0.278v_ct_s \qquad (13)$$

Where:

v_c = *Design speed of cyclist (km/h)*
w_c = *Width of design vehicle (m)*

Under certain circumstances, to avoid a collision, the onus will be on the cyclist to make an emergency stop.

For example, when the cyclist is seen by the driver while still within the minimum stopping distance, regardless of the cyclist's reaction, the driver will be able to come to a stop. However, if the cyclist is spotted when it is too late for the driver to stop, as is the case when the driver enters the triangle before the cyclist, the cyclist is forced to react to the driver and stop instead. Therefore, the cyclist must always have enough distance to react and come to a stop if when the observe an approaching driver.

The emergency stopping distance for a cyclist is given by:

$$D_{ec} = \frac{v_c^2}{25.92a_{ce}} + 0.278v_c t_{rc} \qquad (14)$$

Where:

v_c = *Cyclist design speed (km/h)*
a_{ce} = *Cyclist emergency deceleration rate – gradient adjusted [±9.81G] (m/s²)*
t_{rc} = *Cyclist reaction time (sec)*
G = *Gradient (m/m)*

For design purposes, to ensure the safety of cyclists under all situations, $\mathbf{D_{y2}}$ should be based on the maximum of $\mathbf{D_{nc}}$ and $\mathbf{D_{ec}}$.

$$D_{y2} = \max(D_{nc}, D_{ec} + d_{ec}) + 0.5w_c \qquad (15)$$

Where:

d_{ec} = *Distance from front tire of bicycle to cyclist's eye*
w_c = *Width of design vehicle (m)*

Application

i) Case Study

A 3 meter wide MUP crosses a right-in/right-out driveway from a commercial plaza. The MUP is about 3 m from the edge of the road i.e. X_0 = 6 m (distance from edge + width).

Figure 6: Aerial View : MUP across Commercial Access (Source: York Region, 2019)

Figure 7: Aerial View : MUP across Commercial Access (Source : Google Maps, 2019)

In **Figure 7** it is evident that the MUP is not very conspicuous and there are no additional signage and markings to warn drivers to expect cyclists to travel in the boulevard.

ii) Parameter Assumptions

Table 1 outlines the values of the parameters that were assumed in this case study. Even though best efforts were made to find values that are supported by guidelines, manuals and academic studies, these values are presented for illustrative purposes only and it is not the authors' intent to recommend that these values should be used. It is up to each jurisdiction, who may decide to apply the equations and proposed methodologies in this paper, to select their own parameter values.

Table 1 : Assumed Values of Parameters used in Analysis and Case Studies

Parameter	Value	Description
a_2	1.25 m/s^2	Normal vehicle deceleration rate (Field testing & Maurya & Bokare, 2012)
a_1	1.25 m/s^2	Normal vehicle acceleration rate (Field testing)
a_e	3.4 m/s^2	Emergency vehicle deceleration rate (TRB, 1997)
a_{ce}	2.8 m/s^2	Emergency cyclist deceleration rate (FHWA, 2004)
V_c	15 km/h	Cyclist design speed on MUP (Multi-use path)
t_c	2.5 sec	Cyclist reaction time (TRB, 1997)
t_r	1.5 sec	Reaction time of decelerating driver (Queensland Government, 2016)
l_c	5 m	Length of design vehicle
w_c	2 m	Width of design vehicle
d_s	0.5 m	Separation between stopped car's front bumper and edge of cycling facility/sidewalk
d_o	2.3 m	Distance from front bumper of vehicle to driver's eye
d_{ec}	0.5 m	Distance from front tire of bike to cyclist's eye
w_{cf}	3.0 m	Width of MUP (Multi-use path)

iii) Analysis

Figure 12 was produced by a custom spreadsheet and shows $\mathbf{D_{x1}}$, $\mathbf{D_{y1}}$, $\mathbf{D_{x2}}$ and $\mathbf{D_{y2}}$ as a function of the offset (X_0).

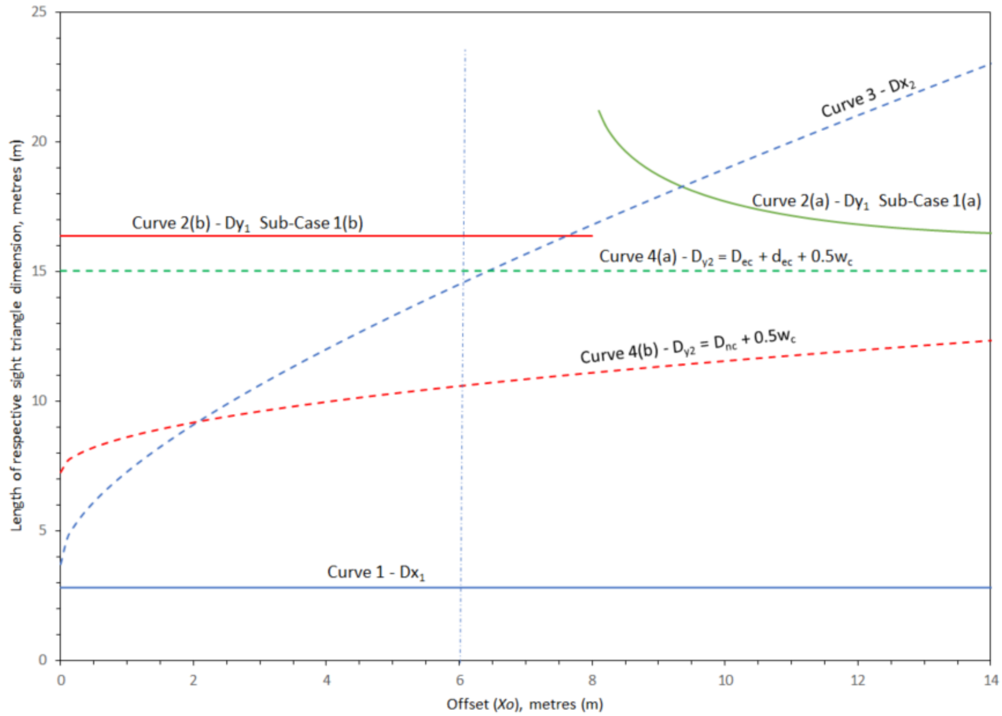

Figure 8 : D_{x1}, D_{y1}, D_{x2} and D_{y2} vs. X_0

Curve 1: This curve describes $\mathbf{D_{x1}}$ from ***Equation 1***.

Curve 2(a): This curve describes $\mathbf{D_{y1}}$ for Sub-Case 1(a) from ***Equation 2,*** with clearance time $\mathbf{t_{cl}}$ calculated using **Equation 7**. If for a particular case this curve falls below Curve 4(a) then $\mathbf{D_{y1}}$ should be determined from Curve 4(a) instead.

Curve 2(b): This curve describes $\mathbf{D_{y1}}$ for Sub-Case 1(b) from ***Equation 2***, with clearance time $\mathbf{t_{cl}}$ calculated using ***Equation 5***. If for a particular case this curve falls below Curve 4(a) then $\mathbf{D_{y1}}$ should be determined from Curve 4(a) instead.

Curve 3: This describes $\mathbf{D_{x2}}$ from ***Equation 8***.

Curve 4(a): This curve describes $\mathbf{D_{y2}}$ from ***Equation 15*** assuming $D_{nc} < D_{ec} + d_{ec}$. This curve can only be used to determine $\mathbf{D_{y2}}$ where it lies 'above' Curve 4(b). In this particular case study all of the curve lies above Curve 4(a) and should it be used to determine $\mathbf{D_{y1}}$ for all possible values of $\mathbf{X_o}$.

Curve 4(b): This curve describes $\mathbf{D_{y2}}$ from **Equation 15** assuming $D_{nc} > D_{ec} + d_{ec}$. This curve can only be used to determine $\mathbf{D_{y2}}$ where it lies 'above' Curve 4(a). In this particular scenario all of the curve lies below Curve 4(a) and should it NOT be used to determine $\mathbf{D_{y1}}$.

For X_o = 6 from **Figure 8** the following values can be determined:

Table 2 : Sight Triangle Dimensions

Dimension	Value
D_{x1}	2.8 m
D_{y1}	16.4 m
D_{x2}	14.5 m
D_{y2}	15.0 m

In **Figure 9** the Case 1 (blue) and Case 2 (red) daylight triangles have been plotted on the aerial image, to scale.

Figure 9: Minimum and Desirable Sight Triangles (Source: York Region, 2019)

It is evident that the minimum sight triangles in this case study is unencumbered, however the desirable sight triangle is obstructed by the corner of a building.

If a high degree of non-compliance with the stop sign, and/or a high volume of cyclists becomes a concern then there are two possible measures that can be implemented – either individually or in combination with each other:

1. Increase the degree of compliance by making drivers more aware that there is a MUP in boulevard that could be used by cyclists – through the effective use of traffic calming, road signage, pavement markings and surface treatments.
2. Decrease the size of the required Case 2 daylight triangle by 'bending' the MUP closer to the roadway.

Table 3 compares the dimensions of the desirable (Case2) triangle before and after the MUP has been 'bend-in' 2 meters closer to the edge of the main road.

Table 3 : Comparison of Case 2 Sight Triangle Dimensions

Dimension	Before	After
X_o	6 m	4 m
D_{x2}	14.5 m	12.0 m
D_{y2}	15.0 m	15.0 m

Moving the MUP closer to the roadway by a distance of 2 meters decreases the magnitude of D_{x2} by 2.5 meters. This decrease, combined with a shift of 2 meters, gives a total gain of 4.5 meters.

Figure 10 shows the before (red) and after (green) Case 2 daylight triangles to the right.

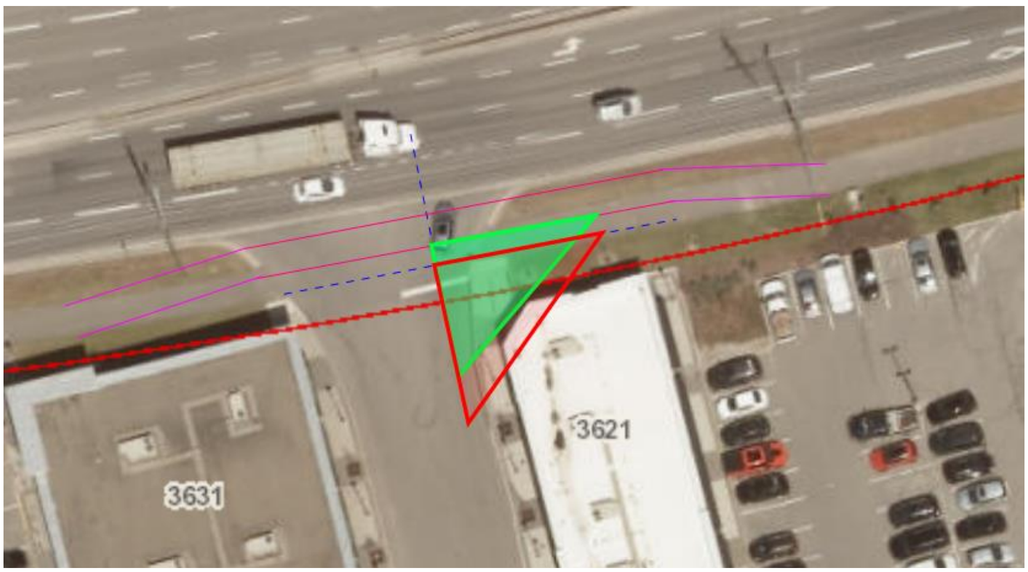

Figure 10: Minimum and Desirable Sight Triangles (Source: York Region, 2019)

From **Figure 10** it is evident that bending the MUP closer to the roadway resulted in a desirable daylight triangle that is no longer obstructed by the building.

Development Planning Case Study

Consider a Site Plan application for a small townhouse development on a Regional arterial as depicted in **Figure 11**. There is an existing 1.5 meter sidewalk which will be upgraded to a 3.0 meter wide MUP as part of a future road reconstruction project. The location of the curb will remain unchanged.

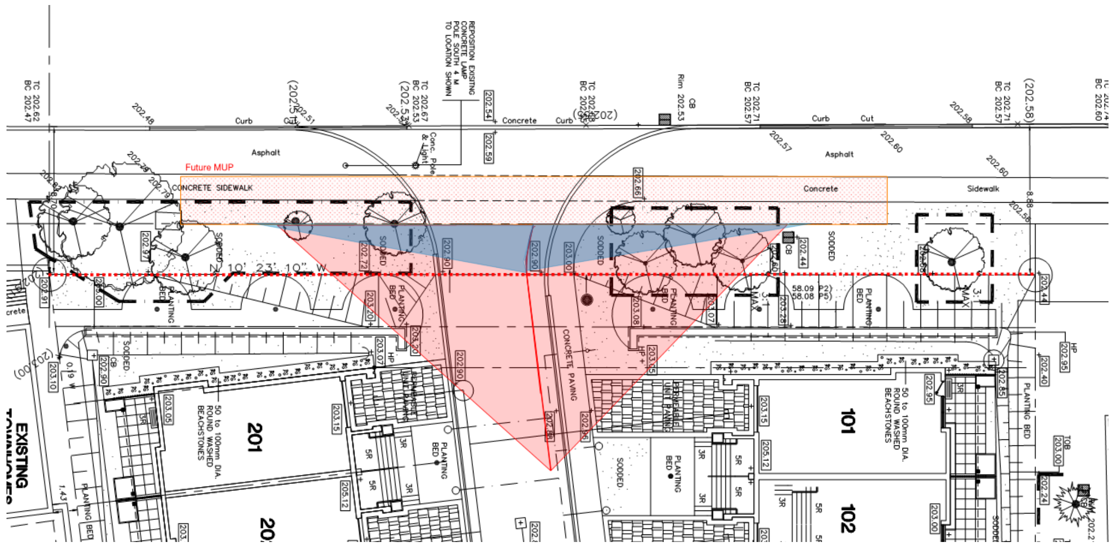

Figure 11: Minimum and Desirable Sight Triangles (Source: York Region, 2020)

In this case X_0 = 6 meters and the values in **Table 2** also applies to this example. **Figure 11** shows the minimum (blue) and desirable (red) sight triangles.

Site Plan approval authorities have the ability to control what the Developer implements within desirable (Case 2) triangles, and can so ensure that no landscape features, that could obstruct site lines within this area, are implemented by the Developer.

Fortunately in this case no buildings are encroaching into the desirable triangle. Attention should be given to the location, size of existing and proposed trees, the height of the wall and the mature height of any plantings proposed in the triangle.

Conclusions

This paper has developed equations and a methodology that can be used to calculate the dimensions of the sight triangles where cycling facilities cross stop-controlled private driveways and public roads – for two types of driver behavior – the 'compliant' driver that stops in advance of the cycling facility and the 'non-compliant' driver that does not stop in advance of the cycling facility. They also take into account the impact of critical design variables, such as the type of facility, and the distance that it is located from the edge of the main road. These equations and the methodology can be used in a retrofit setting to address safety and operational concerns, and in a planning setting to design facilities and site plans that will minimize the risk to drivers and cyclists.

Acknowledgments

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Appendix A

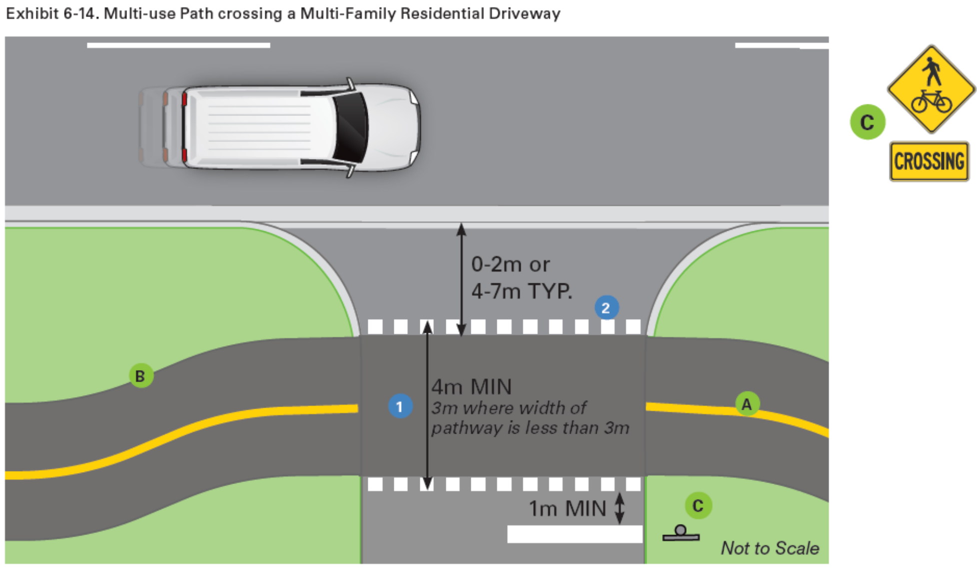

Figure A1 : Bend-in Design – MUP (Source: York Region, 2018)

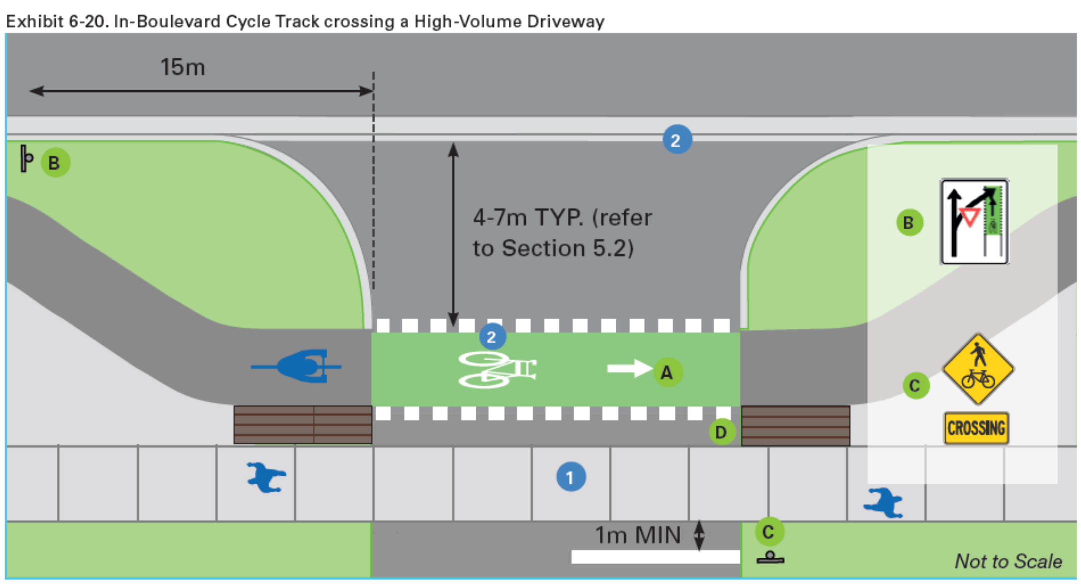

Figure A2 : Bend-out Design – Cycle Track (Source: York Region, 2018)

Appendix B

Calculating the clearance time

Refer to **Figures 3** and **4**.

The total distance the vehicle will travel from start to stop is given by $\mathbf{D_T}$:

$$D_T = X_0 + d_s \qquad \text{(B1)}$$

However, to clear the cycling facility the vehicle only has to travel a distance sufficient to get the back of the vehicle to clear the cycling facility. This clearance distance $\mathbf{D_{cl}}$ is given by:

$$D_{cl} = w_{cf} + d_s + l_c \qquad \text{(B2)}$$

Where:

w_{cf} = *Width of cycling facility (m)*
d_s = *Distance from the front bumper car to the closest edge of the cycling facility (m)*
l_c = *Length of design vehicle (m)*

If the vehicle accelerate at rate $\mathbf{a_1}$ for time $\mathbf{t_1}$ over distance $\mathbf{d_1}$ and decelerate at a rate $\mathbf{a_2}$ to a stop over time $\mathbf{t_2}$ and distance $\mathbf{d_2}$, the maximum speed $\mathbf{v_{max}}$ is given by:

$$v_{max} = a_1 t_1 = a_2 t_2 \qquad \text{(B3)}$$

Where:

a_1 = *Normal acceleration rate - gradient adjusted [±9.81G] (m/s^2)*
a_2 = *Normal deceleration rate - (m/s^2)*
G = *Gradient (m/m)*

Solving for $\boldsymbol{t_2}$ gives:

$$t_2 = \frac{a_1}{a_2} t_1 \qquad \text{(B4)}$$

and

$$D_T = d_1 + d_2 \qquad \text{(B5)}$$

From Newton's 2nd equation of motion $\mathbf{d_1}$ and $\mathbf{d_2}$ can be determined:

$$d_1 = 0.5 a_1 t_1^2 \quad \text{and } d_2 = 0.5 a_2 t_2^2 = 0.5 \frac{a_1^2}{a_2} t_1^2 \qquad \text{(B6 and B7)}$$

Therefore:

$$D_T = 0.5 a_1 t_1^2 + 0.5 \frac{a_1^2}{a_2} t_1^2 \qquad \text{(B8)}$$

Solving for $\mathbf{t_1}$ and then $\mathbf{t_2}$ gives:

$$t_1 = \sqrt{\frac{2D_T}{a_1 + {a_1^2}/{a_2}}} \text{ and } t_2 = \frac{a_1}{a_2}\sqrt{\frac{2D_T}{a_1 + {a_1^2}/{a_2}}} \qquad \text{(B9 \& B10)}$$

The total time to travel for start to stop $\mathbf{t_T}$ is given by:

$$t_T = t_1 + t_2 = (1 + \frac{a_1}{a_2})\sqrt{\frac{2D_T}{a_1 + {a_1^2}/{a_2}}} \qquad \text{(B11)}$$

Distance travelled while accelerating is d_1

$$d_1 = 0.5 a_1 t_1^2 \qquad \text{(B12)}$$

Now there are now two scenarios to consider.

Scenario 1 : $\mathbf{d_1 > D_{cl}}$

In this scenario the back of the car clears the facility while it is still accelerating. Therefore $\boldsymbol{t_{cl}}$ is given by the time the car will accelerate over a distance $\boldsymbol{D_{cl}}$:

$$t_{cl} = \sqrt{\frac{2D_{cl}}{a_1}} \qquad \text{(B13)}$$

Substituting ***Equation B9*** into ***Equation B12*** gives:

$$d_1 = \frac{D_T}{1 + \frac{a_1}{a_2}} \qquad \text{(B14)}$$

Substituting ***Equation B1*** into ***Equation B14*** and re-arranging then gives the range of $\boldsymbol{X_o}$ for which ***Equation B13*** applies:

$$X_o \geq \left(\frac{a_1}{a_2} + 1\right) D_{cl} - d_s \qquad \text{(B15)}$$

Scenario 2 : $\mathbf{d_1 < D_{cl}}$

In this case the back of the car clears the facility after it has reached its maximum speed and is decelerating. Therefore $\mathbf{t_{cl}}$ is given by the time the car will accelerate over a distance $\boldsymbol{D_{cl}}$:

$$t_{cl} = t_T - t_\Delta \qquad \text{(B16)}$$

Where $\boldsymbol{t_\Delta}$ is the time to decelerate at rate $\mathbf{a_2}$ to a stop over a distance of D_T - D_{cl}

$$t_\Delta = \sqrt{\frac{2(D_T - D_{cl})}{a_2}} \qquad \text{(B17)}$$

Therefore

$$t_{cl} = \left(1 + \frac{a_1}{a_2}\right)\sqrt{\frac{2D_T}{a_1 + {a_1^2}/{a_2}}} - \sqrt{\frac{2(D_T - D_{cl})}{a_2}} \qquad \text{(B18)}$$

Appendix C

Calculating the emergency stopping distance (D_e) and time (t_e) to make an emergency stop

Refer to **Figure 5.**

Derived from Newton's 3rd equation of motion, for the driver stopping at the curb :

$$V_i^2 = 25.92a_2(D_e + X_0) \qquad \text{(C1)}$$

For a driver doing an emergency stop:

$$V_i^2 = 25.92a_e(D_e) \text{ and} \qquad \text{(C2)}$$

Where:

- a_2 = *Normal deceleration rate - gradient adjusted [±9.81G] (m/s²)*
- a_e = *Emergency declaration rate - gradient adjusted [±9.81G] (m/s²)*
- X_0 = *The distance between curb and far edge of cycling facility (m)*
- D_e = *Distance to make an emergency stop (m)*
- V_i = *Speed at commencement of emergency stopping (km/h)*

Equating ***Equations C1*** and ***C2*** and then solving for $\mathbf{D_e}$ gives:

$$D_e = \frac{X_0 a_2}{a_e - a_2} \qquad \text{(C3)}$$

Derived from Newton's 2nd equation of motion, the time taken to perform an emergency stop is given by:

$$t_e = \sqrt{\frac{2D_e}{a_e}} = \sqrt{\frac{2X_0 a_2}{a_e(a_e - a_2)}} \qquad \text{(C4)}$$

Calculating distance travelled during perception-reaction time period t_r (D_r)

Over the duration of the perception-reaction time period the driver is decelerating at a normal deceleration rate of $\mathbf{a_2}$.

Therefore by applying Newton's 2nd equation of motion $\mathbf{D_r}$ can be determined by:

$$D_r = 0.278V_i t_r + 0.5a_2 t_r^2 \qquad \text{(C5)}$$

Where:

- t_r = *Driver's perception reaction time (sec)*
- a_2 = *Driver's normal deceleration rate - gradient adjusted [±9.81G]* (m/s2)

Substituting $\mathbf{V_i}$ into ***Equation C5*** gives:

$$D_r = \sqrt{2a_e D_e}\, t_r + 0.5a_2 t_r^2 \qquad \text{(B7)}$$